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Problem Set 6
Due: June 19, 2007, 9:14 am

Embedded Systems

Problem 1 (Reliability) [30 points]

Let $R(t)$ denote the realiability of a system. If the derivative $R'(t)$ exists, and $R(t) \neq 0$, then the FAILURE RATE $r(t)$ is defined by

$$r(t) := \frac{-R'(t)}{R(t)}\,.$$

For instance, if $R(t) = e^{-\lambda t}$ then

$$r(t) = \frac{-(e^{-\lambda t})(-\lambda)}{e^{-\lambda t}} = \lambda\,.$$

(a) For a given system, let $R(t) = e^{-\lambda t^{\alpha}}$, with $\alpha > 1$. What is the failure rate $r(t)$? **[10 points]**

(b) For a given system, let $R(t) = e^{-\lambda t}$ with $\lambda = 10^{-6}\frac{1}{\text{hour}}$. What is the probability that the system functions correctly for at least 1 year? **[10 points]**

(c) For a given system, let $R(t) = e^{-\lambda t^{\alpha}}$ with $\lambda = 5 \cdot 10^{-6}\frac{1}{\text{hour}^2}$ and $\alpha = 2$. What is the probability that the system functions correctly for at least 1 year? **[10 points]**

Problem 2 (Reliability Analysis) [30 points]

Figure 1 shows the reliability diagram of a system. The numbers in the blocks give the reliability of the pertaining system component.

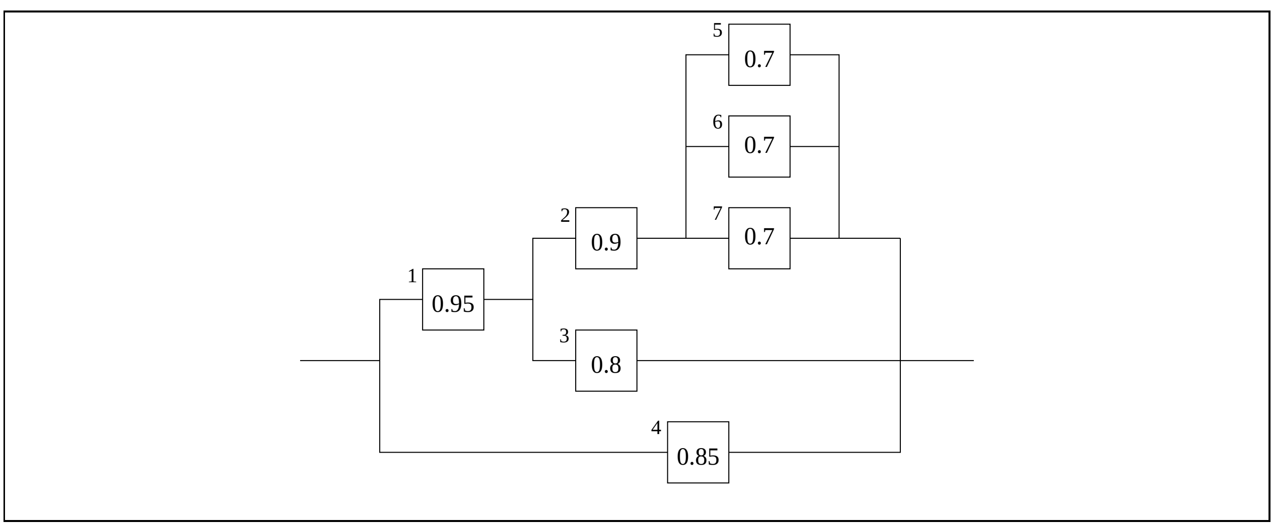

Figure 1: Reliability diagram.

(a) Compute the reliability of the whole system. Show the computation and all partial results. **[10 points]**

(b) A MINIMAL CUT SET is a minimal set of components whose collective failure causes the failure of the whole system. Minimal means that the failure of all components of a proper subset of the cut set does *not* cause the failure of the whole system.

Determine all the minimal cut sets of the given system. Give the minimal cut sets by listing the identifying digits of the components belonging to the cut sets. **[10 points]**

(c) Given all minimal cut sets of a system allows one to determine a lower bound for the reliability of the system. One has:

$$R(t) > 1 - \sum_j \prod_i \left[1 - R_i(t)\right] ,$$

where j ranges over all cut sets, and i ranges over all componenents of the jth cut set. Compute the bound and compare it with the exact bound ascertained in part (a). **[10 points]**

Problem 3 (Static Redundancy) [40 points]

Figure 2 shows the *triple modular redundancy* arrangement described during the lecture. It uses three identical parallel working modules, and works correcly as far as at least two of the three modules are intact.

(a) Assume that all three modules have the same reliability $R(t)$. Also assume that the voter is absolutely reliable. Prove that under these assumptions the reliability of the TMR-arrangement is

$$R_{TMR}(t) = 3R^2(t) - 2R^3(t)$$

[10 points]

(b) Is the realiability of the TMR-arrangement always higher than that of a single module? Justify your answer with a concrete example. **[10 points]**

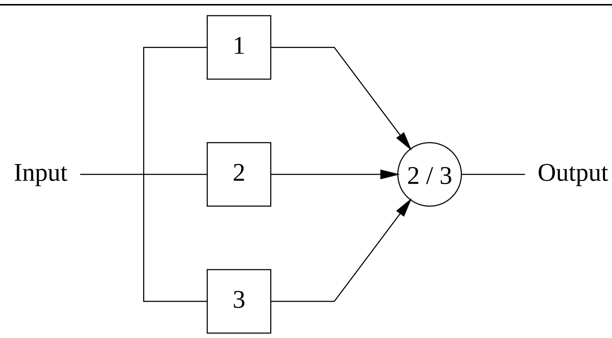

Figure 2: TMR-arrangement.

(c) When a module has a constant failure rate λ then its reliability falls exponentially according to the law $R(t) = e^{-\lambda t}$. At what point of time is the deployment of the TMR versus one single module no more justified? **[10 points]**

(d) Figure 3 shows the reliability diagram of a TMR-arrangement. Derive the equation for the realiability of the TMR by using the same method employed in Problem 2a. Why is the result different from the equation given in Problem 3a? **[10 points]**

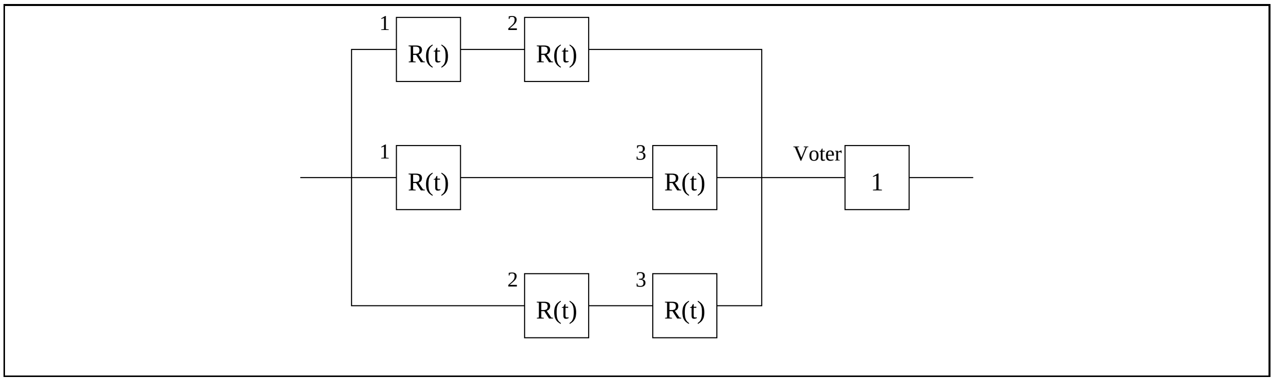

Figure 3: Reliability diagram of a TMR-arrangment.