Embedded Systems

Assignment 6

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6.1 Reliability

a. Let $R(t) = e^{-\lambda t^\alpha}$, $\alpha > 1$.
With this, we can calculate the first derivative of $R(t)$:

$$R'(t) = \left(e^{-\lambda t^\alpha}\right) \cdot \left(-\lambda \cdot \alpha \cdot t^{\alpha-1}\right)$$

Having this, the calculation of the failure rate $r(t)$ is easy:

$$\begin{aligned} r(t) &= \frac{-R'(t)}{R(t)} \\ &= \frac{-\left(e^{-\lambda t^\alpha}\right) \cdot (-\lambda \cdot \alpha \cdot t^{\alpha-1})}{e^{-\lambda t^\alpha}} \\ &= \alpha \cdot \lambda \cdot t^{\alpha-1} \quad \text{for } \alpha > 1 \end{aligned}$$

b. The system should function at least for one year, so
$t = 1a = 24 \cdot 365\ hour$.

$$\begin{aligned} R(1a) &= e^{-\lambda t} \\ &= e^{-10^{-6} \frac{1}{hour} \cdot 24 \cdot 365\ hour} \\ &= e^{-10^{-6} \cdot 24 \cdot 365} \\ &= 0.99128 \end{aligned}$$

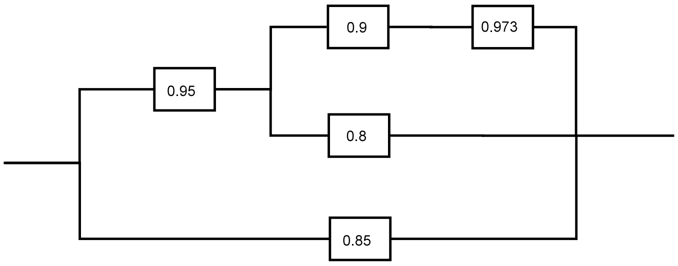
c.

$$\begin{aligned} R(1a) &= e^{-\lambda t^\alpha} \\ &= e^{-5 \cdot 10^{-6} \frac{1}{hour^2} \cdot (24 \cdot 365\ hour)^2} \\ &= e^{-5 \cdot 10^{-6} \cdot 8760^2} \\ &= 0 \end{aligned}$$

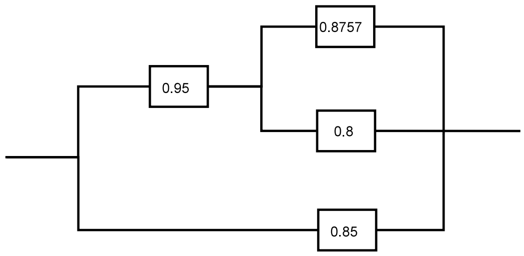
6.2 Reliability Analysis

a.

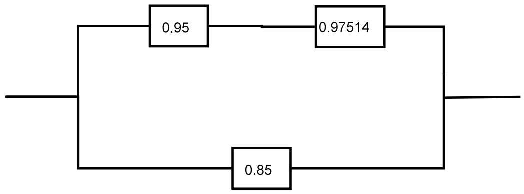
- $R_P = 1 - \prod_{i=5}^{7}(1 - R_i) = (1 - (1 - 0.7)^3) = 0.973$



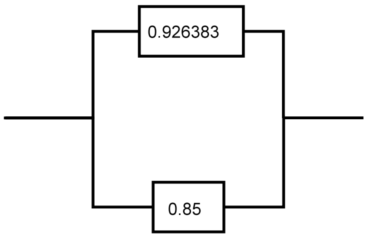
- $R_S = 0.9 \cdot 0.973 = 0.8757$



- $R_P = 1 - ((1 - 0.8757) \cdot (1 - 0.8)) = 0.97514$



- $R_S = 0.95 \cdot 0.97514 = 0.926383$



- $R_P = 1 - ((1 - 0.926383) \cdot (1 - 0.85)) = 0.988957$

b. Minimal Cut Sets:

$$\{1, 4\}, \{2, 3, 4\}, \{3, 4, 5, 6, 7\}$$

c. Using the given formula, we're getting:

$$
\begin{aligned}
R(t) &> 1 - \big((1 - R_1(t)) \cdot (1 - R_4(t)) + (1 - R_2(t)) \cdot (1 - R_3(t)) \cdot (1 - R_4(t)) \\
&\qquad +(1 - R_3(t)) \cdot (1 - R_4(t)) \cdot (1 - R_5(t)) \cdot (1 - R_6(t)) \cdot (1 - R_7(t))\big) \\
R(t) &> 1 - \big((1 - 0.95) \cdot (1 - 0.85) + (1 - 0.9) \cdot (1 - 0.8) \cdot (1 - 0.85) \\
&\qquad +(1 - 0.8) \cdot (1 - 0.85) \cdot (1 - 0.7) \cdot (1 - 0.7) \cdot (1 - 0.7)\big) \\
R(t) &> 1 - (0.05 \cdot 0.15 + 0.1 \cdot 0.2 \cdot 0.15 + 0.2 \cdot 0.15 \cdot 0.3^3) \\
R(t) &> 1 - 0.0075 - 0.003 - 0.00081 \\
R(t) &> 0.98869
\end{aligned}
$$

The exact bound computed in part (a) was $R(t) = 0.988957 > 0.98869$. The result of the approximation (lower bound) is very close to the actual value. Furthermore the actual reliability $R(t)$ is greater than the result of the approximation, which means that the approximation is indeed a lower bound.

6.3 Static Redundancy

a.

- Let's consider the voter first. It is said that the voter is absolutely reliable, which results in the reliability $R_v = 1$.
- For the whole system, there are two main cases:
  1. all the modules work well: $R^3(t)$
  2. one of the modules doesn't work (with three permutations): $3 \cdot R^2(t) \cdot (1 - R(t))$

Let's put all the things together:

$$\begin{aligned} R_{TMR}(t) &= R_v \cdot (R^3(t) + 3 \cdot R^2(t) \cdot (1 - R(t)) \\ &\overset{R_v=1}{=} R^3(t) + 3 \cdot (R^2(t) - R^3(t)) \\ &= 3R^2(t) - 2R^3(t) \end{aligned}$$

b. To get a concrete example where the reliability of the TMR-arrangement is not higher than that of single module, we do some calculation in form of a comparation:

$$\begin{aligned} 3R^2(t) - 2R^3(t) &> R(t) \\ -2R^3(t) + 3R^2(t) - R(t) &> 0 \\ -2R(t) \cdot \left(R^2(t) - \frac{3}{2}R(t) + \frac{1}{2}\right) &> 0 \\ \overset{R(t)>0}{\Longrightarrow} R^2(t) - \frac{3}{2}R(t) + \frac{1}{2} &> 0 \\ R^2(t) - \frac{3}{2}R(t) + \frac{9}{16} &> \frac{1}{16} \\ \left(R(t) - \frac{3}{4}\right)^2 &> \frac{1}{16} \\ \left|R(t) - \frac{3}{4}\right| &> \frac{1}{4} \\ R(t) - \frac{3}{4} > \frac{1}{4} &\vee R(t) - \frac{3}{4} < -\frac{1}{4} \\ R(t) > 1 &\vee R(t) < \frac{1}{2} \end{aligned}$$

Since $R(t) > 1$ makes no sense for a reliability, the calculation gives us, that if the reliability $R(t) < \frac{1}{2}$, the reliability of the TMR-arrangement is not higher than that of a single module. To finally state a concrete example (as expected) we choose

$$R(t) = \frac{1}{8}$$

c. Let $R(t) = e^{-\lambda t}$ and let λ be constant. Let's calculate $R_{TMR}(t)$:

$$\begin{aligned} R_{TMR}(t) &= 3 \cdot \left(e^{-\lambda t}\right)^2 - 2 \cdot \left(e^{-\lambda t}\right)^3 \\ &= 3 \cdot e^{-2\lambda t} - 2 \cdot e^{-3\lambda t} \\ &= e^{-2\lambda t}\left(3 - 2 \cdot e^{-\lambda t}\right) \end{aligned}$$

Now we want to have a look at the time t where the deployment of the TMR versus one single module is no more justified.

$$\begin{aligned} R_{TMR}(t) &= R(t) \\ e^{-2\lambda t} \cdot \left(3 - 2 \cdot e^{-\lambda t}\right) &= e^{-\lambda t} \\ 3 - 2 \cdot e^{-\lambda t} &= e^{\lambda t} \\ 3 &= e^{\lambda t} + 2 \cdot e^{-\lambda t} \\ 3 \cdot e^{\lambda t} &= \left(e^{\lambda t}\right)^2 + 2 \\ \text{Substitution: } z = e^{\lambda t} & \\ 3z &= z^2 + 2 \\ z^2 - 3z + 2 &= 0 \\ z^2 - 3z + \frac{9}{4} &= \frac{1}{4} \\ \left(z - \frac{3}{2}\right)^2 &= \frac{1}{4} \\ \left|z - \frac{3}{2}\right| &= \frac{1}{2} \\ z - \frac{3}{2} = \frac{1}{2} &\vee z - \frac{3}{2} = -\frac{1}{2} \\ z = 2 &\vee z = 1 \\ \text{Substituting back:} & \\ e^{\lambda t} = 2 &\vee e^{\lambda t} = 1 \\ \lambda t = \ln(2) &\vee \underbrace{\lambda t = \ln(1)}_{\Rightarrow t=0} \; (*) \\ \\ \lambda t &= \ln(2) \\ \Rightarrow t &= \frac{\ln(2)}{\lambda} \end{aligned}$$

Since $t = 0$ in the case $(*)$ doesn't make sense, we only used the other result.

d. Because we think that there is an ambiguity in the text of this part, we offer multiple solutions:

- $R_{TMR'}(t) = R^3(t)$. This is due to the fact that if one module per line fails, the whole line cannot be used. Furthermore, another line is also useless because each module is located in two parallel computations.

- Obviously, we have a parallel arrangement. In each of the parallel lines we have two serial blocks. For the two serial blocks, we get: $R_1(t) \cdot R_2(t)$ for the first line, $R_1(t) \cdot R_3(t)$ for the second line and $R_2(t) \cdot R_3(t)$ for the third line. For the parallel arrangement, this means that:

$$\begin{aligned}
R_P &= 1 - ((1 - R_1(t) \cdot R_2(t)) \cdot (1 - R_1(t) \cdot R_3(t)) \cdot (1 - R_2(t) \cdot R_3(t))) \\
&= 1 - ((1 - R_1(t)R_2(t) - R_1(t)R_3(t) + R_1^2(t)R_2(t)R_3(t)) \cdot (1 - R_2(t)R_3(t))) \\
&= 1 - (1 - R_1(t)R_2(t) - R_1(t)R_3(t) + R_1^2(t)R_2(t)R_3(t) - R_2(t)R_3(t) \\
&\qquad + R_1(t)R_2^2(t)R_3(t) + R_1(t)R_2(t)R_3^2(t) - R_1^2(t)R_2^2(t)R_3^2(t)) \\
&= R_1(t)R_2(t) + R_1(t)R_3(t) - R_1^2(t)R_2(t)R_3(t) + R_2(t)R_3(t) \\
&\qquad - R_1(t)R_2^2(t)R_3(t) - R_1(t)R_2(t)R_3^2(t) + R_1^2(t)R_2^2(t)R_3^2(t))
\end{aligned}$$

- If we furthermore assume that the reliability of $R_1(t)$, $R_2(t)$ and $R_3(t)$ are equal, i.e. $R_1(t) = R_2(t) = R_3(t) = R(t)$, we yield

$$\begin{aligned}
R_P &= R^2(t) + R^2(t) - R^4(t) + R^2(t) - R^4(t) - R^4(t) + R^6(t) \\
&= R^6(t) - 3R^4(t) + 3R^2(t)
\end{aligned}$$